Disentangling crashes from tail events

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May 9, 2012, Paris

Outline

Abstract

The study of tail events has become a central preoccupation for academics, investors and policy makers, given the recent financial turmoil. However, the question on what differentiates a crash from a tail event remains unsolved. This article elaborates a new definition of stock market crash taking a risk management perspective based on an augmented extreme value theory methodology. An empirical test on the French stock market (1968-2008) indicates that it experienced only two crashes in 2007-2008 among the 12 identified over the whole period.

Key Words: Crash, Volatility, Risk Management, Contagion Effect, Systemic Risk.
JEL Classification: C4, G01, G28, G32.

The problem

The question

- What differs a crash from a tail event? Why is this distinction useful?
- What is the role played by volatility?
- How defining a stock market crash from a risk management stand point?

The intuition

- Sudden event
- Significant decline
- Short-term horizon

Literature review

Extreme value theory - risk management perspective

- Longin (1993, 1996, 2001), Embrechts, Kluppelberg and Mikosch (1997), McNeil (1997), Danielsson and de Vries (1997), McNeil and Saladin (1998), McNeil (1999), Coles (2001), Beirlant, Goegebeur, Segers and Teugels (2004), McNeil and Frey (2000), Gilli and Këllezi (2000), Bali and Neftci (2002), Jondeau and Rockinger (2003), LeBaron and Samanta (2004), Tolikas and Brown (2005), Gettingby, Sinclair, Power and Brown (2006) etc
- Longin, Boulier and Dalaud (1998), Robert (1998)

General theories - macro-finance perspective

- Patel and Sarkar (1998), Laloux, Potters, Cont, Aguilar and Bouchaud (1999), Gabaix, Gopikrishnan, Plerou and Stanley (2003), Watanabe, Takayasu and Takayasu (2007), Johansen, Ledoit and Sornette (2008) etc
- Gallais-Hamonno and Arbulu (2002), Le Bris (2008), Le Bris and Hautcoeur (2008)

Tail identification

Graphical selection

- Mean residual life plot
  $\left[u, \frac{1}{n_u} \sum_{i=1}^{u} (x_i - u)\right]$

- Threshold stability plot
  $\left[\xi, G_{\xi,\beta}(x) = 1 - (1 + \xi x/\beta)^{-1/\xi}\right]$

Optimal selection

- Beirlant et al. (2004) propose a criterion for the optimal number of observations in the tail that is based on the Hill (1975) estimator
  $\left[n_u, \widehat{\xi}_{n_u}^{H} = \frac{1}{n_u} \sum_{i=1}^{n_u} (logX_{n-i+1} - logX_{n-n_u})\right]$

Crash definition

Sudden

- A price variation independent of the current volatility regime
- A high-return shock during a period of low volatility

Significant

- A price variation with high magnitude
- A jump in the volatility process caused by the leverage effect
  - Local crash
  - Global crash caused by contagion effect

Short-term

- One-day time period

Crash modelling

Sudden

$$\forall t \in [1, ..., T], Z_t = min[Z_1, ..., Z_{t-1}]$$

Significant

- Jump volatility effect

$$\forall t \in [1, ..., T], \frac{\sigma_{t+1}}{\sigma_t} = max[\frac{\sigma_2}{\sigma_1}, ..., \frac{\sigma_t}{\sigma_{t-1}}]$$

- Contagion effect

$$\forall t \in [1, ..., T], |\rho_{t+1} - \rho_t| = max[|\rho_2 - \rho_1|, ..., |\rho_t - \rho_{t-1}|]$$

Short-term

$$dt = 1/252$$

Data modelling

Return generating process

$$R_t = \mu + \sum_{i=1}^{2} \phi_i R_{t-i} + \sum_{i=1}^{4} \theta_i \epsilon_{t-i} + \epsilon_t$$

Time-varying volatility

$$\sigma_t^2 = \omega + \alpha \left(Z_{t-1}\sigma_{t-1}\right)^2 + \gamma \left(Z_{t-1}\sigma_{t-1}\right)^2 I_{Z_{t-1}\sigma_{t-1}<0} + \beta\sigma_{t-1}^2$$

Return transformation

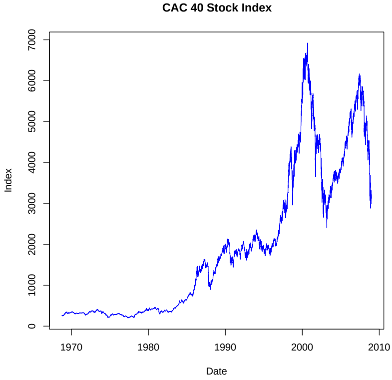
- Innovations $Z_t \hookrightarrow F_Z(0,1)$ are iid with $Z_t = \epsilon_t/\sigma_t$
- Quantile regression for transforming Z into $\widehat{R}$

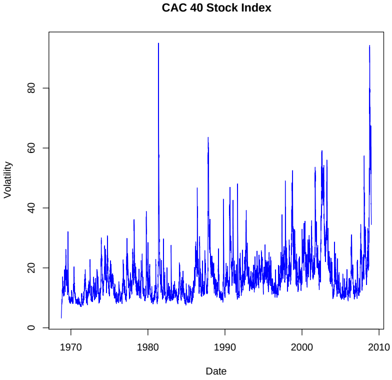
The data

The database

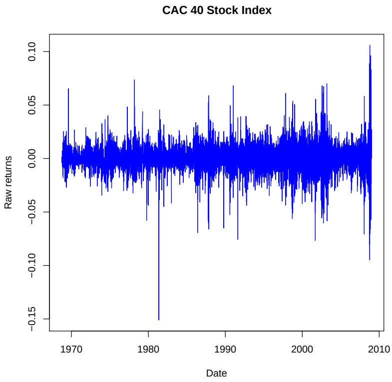
- 10,017 daily stock prices
- From September 30, 1968 to December 31, 2008
- The longest french daily data set available from NYSE-Euronext begins from January 5, 1962
- Equal period for the S&P 500 stock index for contagion measure

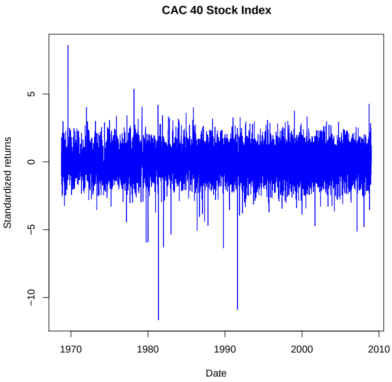
French stock market (1968-2008) I





French stock market (1968-2008) II





Tail identification I

Right tail identification

- $+\widehat{Z} = +0.95\%$
- $+\widehat{\widehat{R}} = +1.28\% \approx +1.5\%$
- 1522 obs.

Left tail identification

- $-\widehat{Z} = -1.38\%$
- $-\widehat{\widehat{R}} = -1.92\% \approx -2.0\%$
- 755 obs.

Tail identification II

Maximum likelihood estimators of the right tail GPD

- $\widehat{\xi} = -0.00055$
- $\widehat{\beta} = +0.5403^{***}$

Maximum likelihood estimators of the left tail GPD

- $\widehat{\xi} = +0.14397^{***}$
- $\widehat{\beta} = +0.5015^{***}$

Extreme downside risk

GPD-VaR at 99%

- $\widehat{Z} = -2.55\%$
- $\widehat{\widetilde{R}} = -3.16\%$

GPD-ES at 99%

- $\widehat{Z} = -3.34\%$
- $\widehat{\widetilde{R}} = -3.99\%$

The maximum crash expected to be exceeded once every century

- $\widehat{Z} = -9.98\%$
- $\widehat{\widetilde{R}} = -10.99\% \approx -11\%$

Crash event identification I

Monthly crashes identified in literature

Date	LB (1)	Date	LB (2)	Date	GH-A
May 1981	-32.79%	May 1981	-4.48%	November 1987	-18.36%
October 1987	-25%	October 1987	-3.49%	June 1981	-14.37%
May 1986	-18.56%	September 1998	-3.23%	October 1987	-14.02%
September 1998	-17.66%	May 1986	-2.89%	May 1981	-12.96%
July 2002	-15.99%	November 1973	-2.85%	August 1990	-12.79%
October 2008	-14.99%	October 2008	-2.79%		
September 2002	-14.81%				
November 1973	-14.31%				
August 1990	-13.39%				
June 1974	-12.82%				

Crash event identification II

The 12 crashes

- 1^{st} biggest crash (1981-05-13)
  Z=-11.66%; σ_{t+1}/σ_t = 4.63; $|\rho_{t+1} - \rho_t|$ = 6.17% #14
- 2^{nd} biggest crash (1991-08-19)
  Z=-10.89%; σ_{t+1}/σ_t = 4.34; $|\rho_{t+1} - \rho_t|$ = 24.01% #3
- 3^{rd} biggest crash (1989-10-16)
  Z=-6.36%; σ_{t+1}/σ_t = 2.64; $|\rho_{t+1} - \rho_t|$ = 6.70% #12
- ...
- 8^{th} biggest crash (2007-02-27)
  Z=-5.12%; σ_{t+1}/σ_t = 2.21; $|\rho_{t+1} - \rho_t|$ = 27.71% #1
- ...
- 10^{th} biggest crash (2008-01-21)
  Z=-4.79%; σ_{t+1}/σ_t = 2.09; $|\rho_{t+1} - \rho_t|$ = NA (memorial day in U.S.)
- ...

Contributions and results

The general contribution

- A new stock market definition

The empirical contributions

- Application to the French stock market (1968-2008)
- Tail areas identification with extreme value theory + quantile regression procedure

The empirical results

- The tail area begins from $+1.5\%$ for the right tail and -2.0% for the left tail.
- The maximum crash, expected to be exceeded once every century, is -11%
- A total of 12 crashes are identified over the 1968-2008 period vs 2 crashes in 2007-2008.