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Risk Management in Insurance

What technical solutions to answer the huge modeling requirements?

WG Risk, 14th December 2011

Insurance: mean, risk premium and VaR99.5%...

- **The insurance business model core: the Risk**
  - The insurer « **purchases risk** » (insurance contracts, financial covers)
  - It « **transforms risk** »:
    - Mutualization (law of large numbers)
    - Diversification
  - It « **resells risk** »:
    - Reinsurance
    - Securitization

Insurance: mean, risk premium and VaR99.5%...

- **Bernoulli example (1738):**
  - *Sempronius owns goods at home worth a total of 4000 ducats and in addition possesses 8000 ducats worth of commodities in foreign countries from where they can only be transported by sea. However, our daily experience teaches us that of [two] ships one perishes*
  - *This wealth may represented by a lottery ~x , which takes on a value of 4000 ducats with probability 1/2 (if his ship is sunk), or 12 000 ducats with probability 1/2 .*
  - **Mean**: *E˜x ≡ ½*4000+½*12000=8000 ducats*
  - *Now Sempronius has an ingenious idea. Instead of "trusting all his 8000 ducats of goods to one ship," he now "trusts equal portions of these commodities to two ships." Assuming that the ships follow independent but equally dangerous routes, Sempronius now faces a more diversified lottery ˜y distributed as (4000, 1/4; 8000, 1/2; 12 000, 1/4).*
  - **Mean**: *E˜y = ¼*4000 + ½*8000 + ¼*12 000 = 8000 ducats*
  - Same **Mean**, means same intrinsic value?

➔**NO!**

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- **Bernoulli example (1738): (2)**
  - Positive value of risk diversification
  - Bernoulli proposed to replace E~x by E(log ~x)
  - More generally, we can replace E~x by E u(~x) = ∑ p*u(x) with u concave
  - Equivalent no risky amount : u(c(˜x)) = E[u(˜x)]
  - c(˜x) < E(˜x) ; the difference between the 2 = **Risk premium**
  - **Insurance tariff can be based on the Mean + Risk Premium**
  - In the real world we have to take account legal issues (eg mandatory insurance), as well as competitive and distribution network aspects

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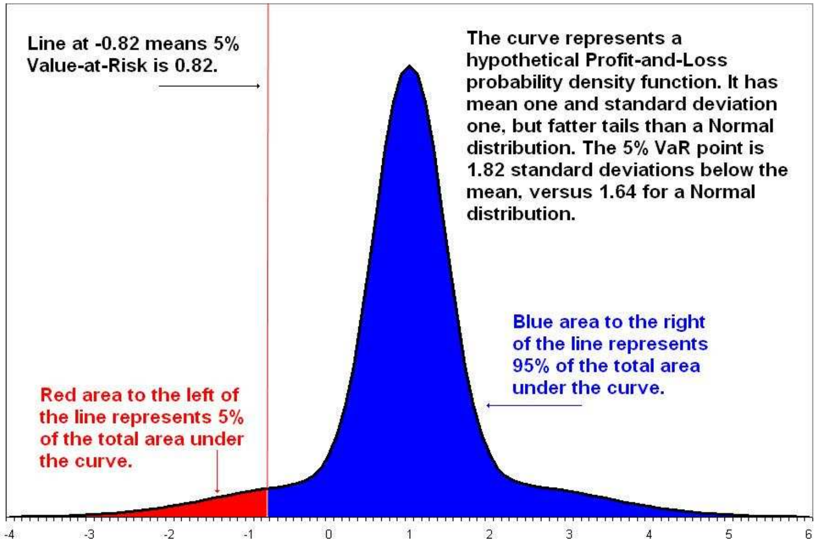
Insurance: mean, risk premium and VaR99.5%...

- **Insurance and capital**
  - **Need of capital**:
    - If insurers keep in their accounts the **Mean** of what they will have to pay (the **Reserves**)

    → 1 chance on 2 that they don't keep enough money, and go bankrupt.

    → insurance authorities impose insurance companies to keep an amount of capital:

    - Until 2013: based on a prorata of **Premiums** and **Reserves** (European **Solvency I** Directive)
    - After 2013: based on a **VaR99.5%** on a 1 year horizon of the overall assets liabilities: the **SCR** (European **Solvency II** Directive)
    - Some insurers (AXA, GENERALI, SCOR…) already had an "**Economic Capital**" valuation model based on an internal model.



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Insurance: mean, risk premium and VaR99.5%...

- **What does it imply regarding modeling?**
  - Pricing: micro-models, the commercial premium (mean + commercial charges) has to be evaluated contract by contract
    - Example of Motor Premiums: evaluated according to pricing factor (age, type of car, localization…) ➔ mostly GLM Models. Segmentation vs Diversification.
  - Reserving: Mean (“Best Estimate”) + Risk Margin.

    Eg Chain Ladder models, use of a_x for annuities + Cost of capital for Risk Margin (European decision in 2007, rather than VaR or TVaR-based calculations)
  - Capital: Use of standard or internal models.

    All risks have to be jointly model to take into account mutualization & diversification effects. VaR99.5% estimation ➔ 200 simulations = minimum, 10 000 simulations = recommended.

Insurance: VaR 99.5% / the modeling challenge

- **Illustration**:
  - 1 distribution compound poisson per contract/client (number of claims / size of each claim)
  - 3 000 000 clients
  - Average of 3 contracts per client (different lines of business: car, life, household)
  - 3 similar companies in the group
  - 10 000 simulations
  - 20 years projection
  - Global model = 2 * 3 000 000 * 3 * 3 * 10 000 * 20 = $1.08*10^{13}$ simulations
  - Insurance modeling software perform 100 000 to 2 000 000 simulations / second
  - **Calculation time : 62 days to 3.5 years!!!!**

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Insurance: VaR 99.5% / the modeling challenge

- **Required simplification**:
  - Separate attritional / large claims :
    - Calibrate a frequency/severity model for each line of business
    - Model large losses only with a Compound Poisson
  - In life: use of model points instead of head by head calculations
  - Eg 2 simulations needed for the attritional per line of business, 40 simulations required in average for large losses
  - Global model = 2 * (2+40) * 3 * 3 * 10 000 * 20 = 151 200 000 simulations
  - *Theoretical Calculation time : 1.3 minute to 23 minutes*
  - **But** many other parameters to take into account: claims payment timing, expenses, assets (modeling, dynamic reinvestments etc…), profit sharing policy
  - Therefore instead of 42 calculations, ACTUARIS non life models have eg around 1 200 elements to be calculated for each line of business
  - Calculation time : 36 minutes to 12 hours: **We are in the range of usability/unusability of models, and this only for 3 lines of business**

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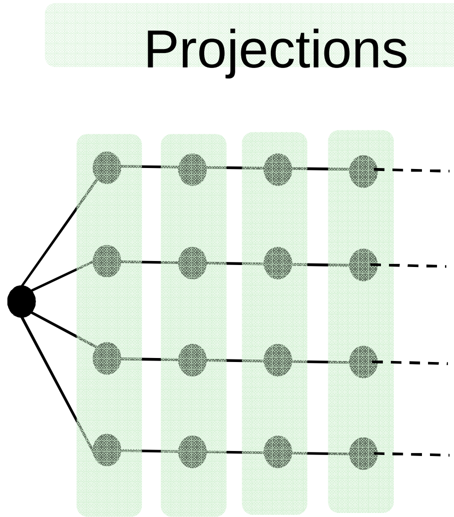
Insurance: VaR 99.5% / the modeling challenge

- **Random generators**:
  - Several billions of calculation required.
  - Problematics with standard random generators:
    - Shorter than expected periods for some seed states (such seed states may be called 'weak' in this context);
    - Lack of uniformity of distribution for large amounts of generated numbers;
    - Correlation of successive values;
    - Poor dimensional distribution of the output sequence;
    - The distances between where certain values occurrence are distributed differently from those in a random sequence distribution.

  ➔ Use of more advanced generators, eg
    - **Mersenne twister** MT19937 (periodicity of 2^19937…) – most used in Finance
    - **Brent Xorgens** XOR4096 (periodicity of 2^4096)
    - These random generators are amongst the best for the 2 reference randomness test: **DieHard** and **U01**

Insurance: VaR 99.5% / the modeling challenge

- **The memory issue**:
  - 4 300 000 000 data to be computed. 32 Go Memory required in this simplified model (8 bytes/number)!
  - Requirement to keep most projections in memory to calculate EV.
  - As an illustration 32 bits computers can only address 2 Go!
  - First possible solution: **Vertical calculations**
  - Enables to calculate immediately risk measure then if the variable is not reused to drop the unused values
  - Issue: requirement to keep all 10 000 simulations of each variable which will be reused in further calculations



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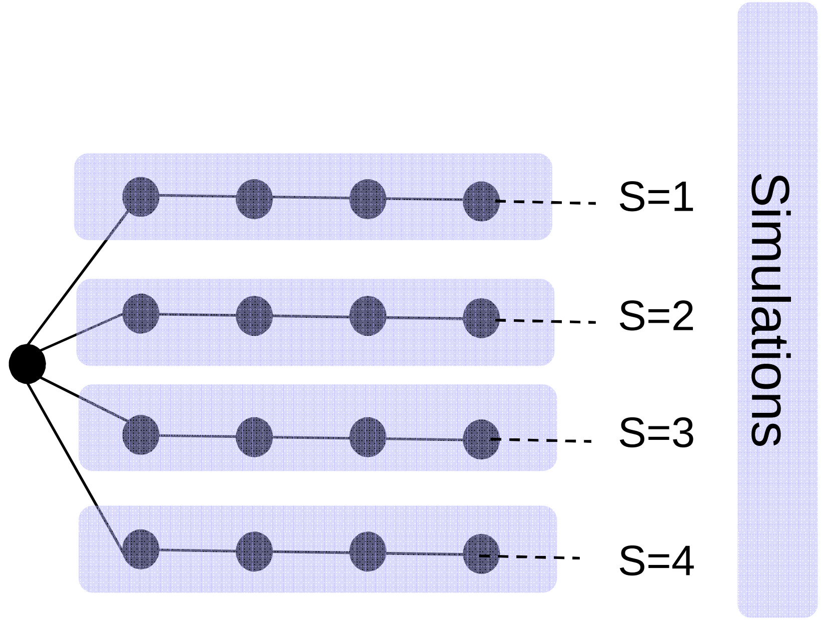
Insurance: VaR 99.5% / the modeling challenge

- **The memory issue (2)**:
  - Second solution: **Horizontal calculations**
  - We have to keep all simulations only for the variables for which we want to calculate the risk measures



  - In any case need of an **"intelligent" memory management** within the software which will design the optimized "route" for running the simulations (objective: using the minimum RAM possible, minimizing the disk access, in consideration of the target)

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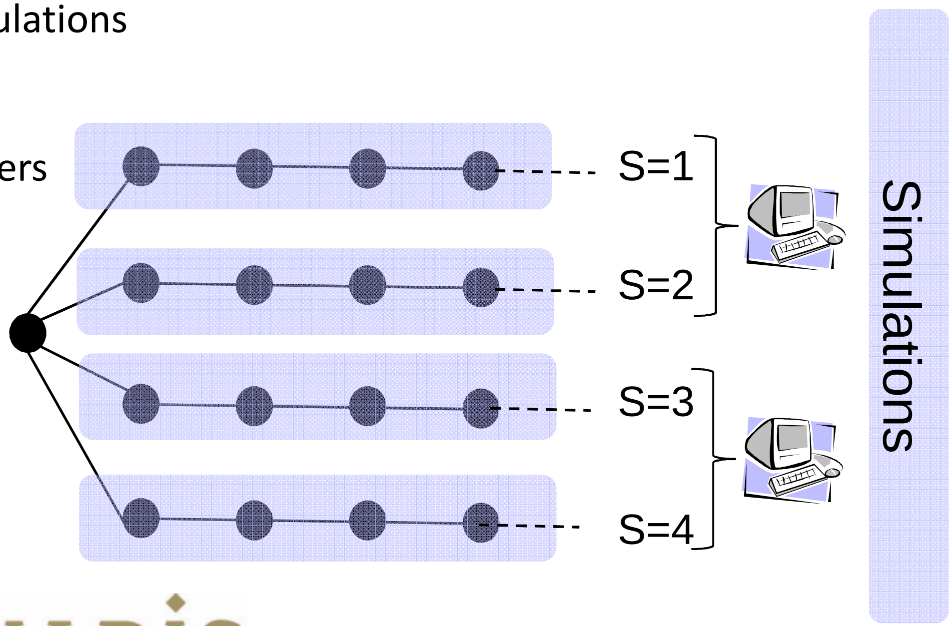
Insurance: VaR 99.5% / the modeling challenge

- **Issue of bigger models** :
  - Most companies do not have only 3 lines of business to model. EG SCOR : >700 lines of business.
  - If possible ➔ Simplification of the model. If not ➔ Need to search for alternative IT solutions:
    - **Multi-core computing**:
      - Use of all cores of the computer
      - Each one performs a set of simulations
    - **Grid computing**
      - Same idea with several computers

        (defined: cluster computing or undefined: grid computing)
      - Each computer performs a set of simulation on all its cores
      - Issue of memory transfer & risk measures calculation



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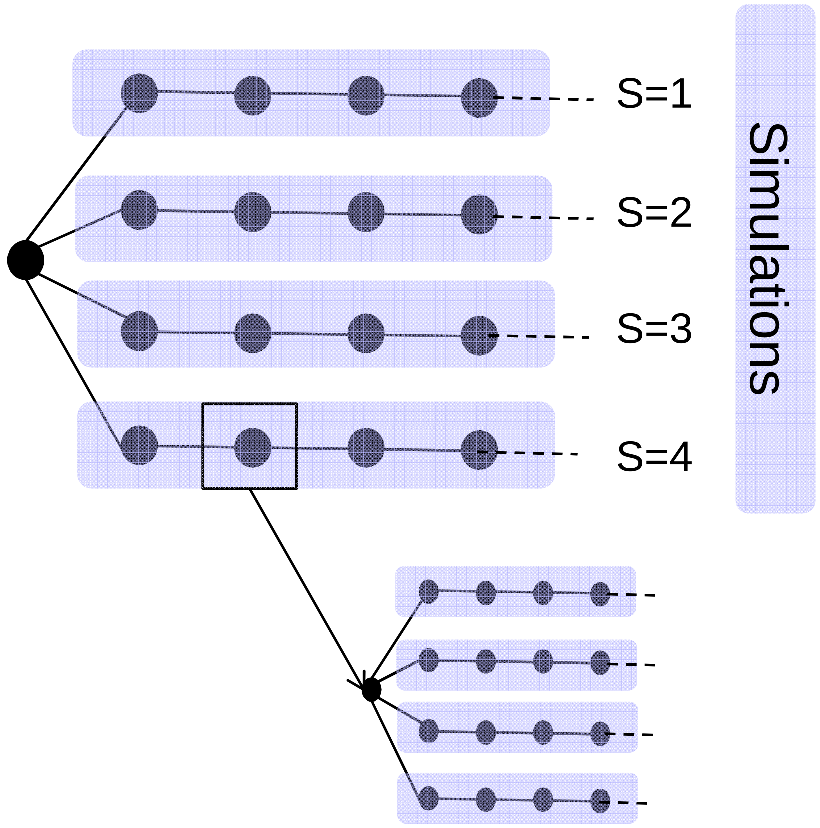
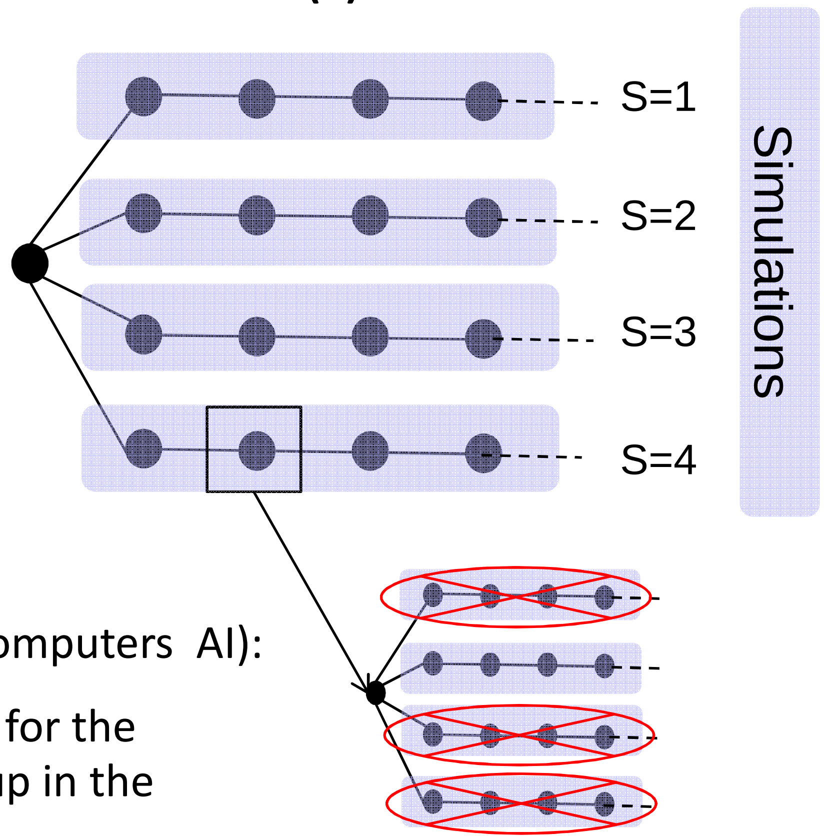
Insurance: VaR 99.5% / the modeling challenge

- **Issue of Life lines of business: "Simulations in Simulations"**
  - The estimate of the one year VaR99.5% implies for each simulation performing a sub-simulation to calculate the best estimate of the liabilities in year 1.
  - This implies theoretically 10 000 * 10 000 simulations.
  - >250 days of calculation time!!!



Insurance: VaR 99.5% / the modeling challenge

- **Issue of Life lines of business: "Simulations in Simulations" (2)**
  - Proposed solutions:
    - Best estimate = Mean. It requires much less calculations than VaR99.5% for a robust result.
    - → Possibility to have 100-1000 simulations to calculate best estimate, then 10 000 simulations to calculate VaR99.5%. But still several days of calculation time…
    - Lines reduction (similar to chess computers AI):
    - → Not calculating the sub-simulation for the simulations very likely not to end up in the 1/200 worst cases.



Insurance: VaR 99.5% / the modeling challenge

- **Summary of proposed solutions:**
  - **Simplification of models**
    - Non life: **separate attritional / large claims**
    - Life: **use of model points**
  - **Memory management**
    - Use **horizontal calculations**
    - Implement an "**intelligent memory management**" to optimize calculation route
  - **Bigger model**
    - Use **multi-core** calculations
    - Implement **grid computing**
  - **Simulations in Simulations**
    - Use less simulations to calculate life Best Estimate in sub-projections
    - Use **lines reduction** techniques

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ERM System & ACTUARIS International

Powerful modeling new generation modeling platform:

- Powerful stochastic engine with **Mersenne Twister** and built-in **copula** mechanism. Average of 2 million simulations per second on a simple computer.
- **Intelligent memory management** mode: adapts model setting to reach a target maximum required RAM
- Automated **multi-core** mode
- **Grid computing** capacities
- **Simulations in simulations** functionality

Available for universities and research centers.
Free for students.

ACTUARIS International:

- Over 80 associates providing actuarial & strategic consulting and software
- 23 countries using ACTUARIS International software
- 200 insurance companies client
- 800 users
- Offices in France (Paris, Lyon), Belgium (Brussels), Spain (Madrid)
- Alliance in Germany (Ulm)

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