Risk measurement and its limits in asset management

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Introduction
> What is a financial asset?

- **A financial asset should provide some positive long-term return:**
  - the investment universe is not risk-neutral, whence the existence of a risk premium (excess return as compared to the risk-free rate) over the long-term
  - in the absence of any risk premium, no durability for an asset as financial investment.
  - the economic model may be questioned temporarily or for good .

- **Common asset classes:**
  - Treasuries
  - Sovereign bonds (developped and emerging countries)
  - Corporate bonds
  - Developed markets and Emerging markets equities
  - Private equity
  - Real estate

- **Beware of pseudo asset classes: where is the long-term premium?**
  - Currencies
  - Volatility
  - Commodities

Introduction
> Aim of asset management

- **The aim is to provide a better long-term performance than risk-free rate :**
  - this comes together with a higher long-term risk.

- **Asset management is a return/risk approach:**
  - maximize return for a given level of risk
  - minimize risk for a given level of return

- **Asset allocation results from optimization:**
  - strategic asset allocation: deals only with real assets in the long-term
  - tactical asset allocation: may deal with other types of investments but should be supported in the short/ mid-term (horizon of a few weeks)

- **1. Forecasting returns:**
  - Macroneconomic analysis (asset class)
  - Fundamental or microeconomic analysis (stock picking)
  - Econometric models (relative value)
  - …

- **2. Managing risk:**
  - What is the nature of risk
  - How to measure risk
  - How to control risk

Introduction
> Sources of financial risk

- **Market risk (asset-class specific):**
  - Equities : economic slow-down, etc.
  - Bonds (sovereign and corporate) : interest rate risk, rating downgrade risk
- **Sector risk:**
  - Debt crisis particularly affecting the (bancaires?)
  - Bubble bursts (TMT in 2000)
- **Specific risk:**
  - company governance (embezzlement eg ENRON failure)
  - industrial hazards (BP, etc.)
- **Liquidity risk:**
  - risk of being unable to sell or value a product on an illiquid market
- **Model risk:**
  - risk of a quantitative strategy which worked in the pats being ineffective in the future (paradigm change)
- **Counterparty risk:**
  - structured products guaranteed by the emitter (Lehman Brothers failure)

Introduction
> Does a risk-free asset really exist?

- **The risk-free rate for a given horizon is that of treasury bonds for the same horizon:**
  - is it really risk-free?
  - for which investors?
  - at which horizon?

- **Currency risk:**
  - American treasury bonds are not risk-free for European investors
  - Some countries (emerging countries) emit mainly in hard foreign currencies in order to be able to borrow.
  - locals therefore do not have a risk-free rate, as they are exposed to a currency risk.

- **Interest-rate risk:**
  - In a 3-month Markowitz optimization, the risk-free rate is that of 3-month government bonds
  - If some event forces a portfolio recomposition after 1 month, 3-month bonds are no longer risk-free.
  - It may be that the rate at the desired horizon does not exist.

- **Counterparty risk:**
  - If the central bank does not monetize the debt, a default might occur on the government bonds.
  - What is the risk-free rate in the eurozone?

Volatility as measure of risk
> Is it a good measure of risk?

- **Most common measure of portfolio risk :**
  - Simple to use
  - Sharpe ratio = excess return/vol is the most popular measure of the performance (Morningstar)

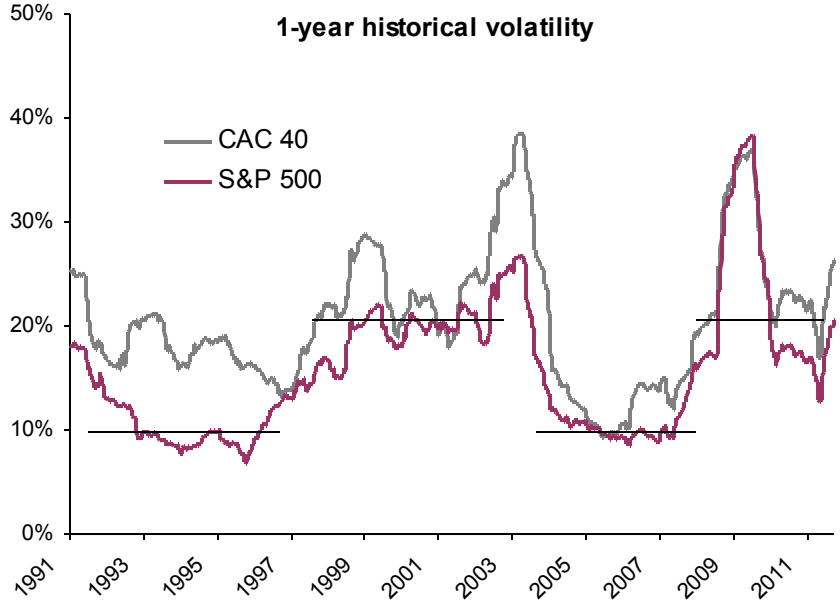
- **Diversification as a consequence**
  - Let two assets have expected return r and volatility σ
  - The volatility of an equal-weighted portfolio is $\sigma[(1+\rho)/2]^{1/2} < \sigma$ (while the expected return is r)

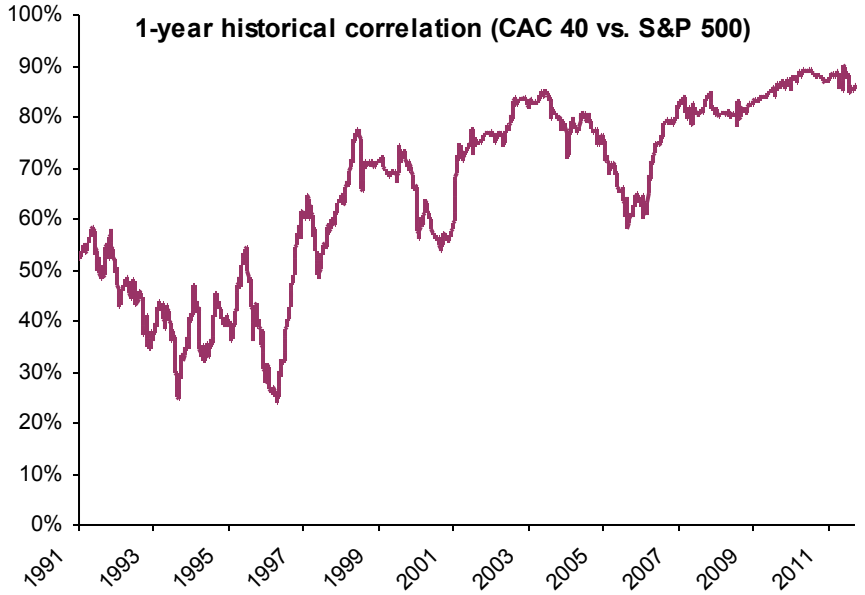
- **Hypothesis H : asset returns are Gaussian and stationay (or iid)**
  - It is sufficient to determine expected return of each asset and the correlation matrix
  - Historical volatility is an estimator of (true) volatility

- **But H is generally false :**
  - Non-Gaussian returns : volatility is not sufficient to determine risk
  - Non-stationary returns : problem of estimation of volatily

Limits of volatility
> 1st limit : non stationary returns

Historical correlations and volatilities :





Stationarity hypothesis is usually rejected :

- Historical volatility is not an estimator of volatility
- Volatility is no more a constant

We suppose correlation matrix to be constant over short period of time for some assets :

- The estimation is done over short periods
- There are 2 regimes for common assets (Bullish and bearish markets) :

Limits of volatility
> 2nd limit : problem of estimation

- **Correlation matrix estimation is done over 1 to 3 years (cf 1st limit)**
  - The historical correlation matrix may be very unstable
  - In the CAC 40 universe, the matrix correlation is 40x40 ⟶ 40x41/2=820 parameters to estimate

- **(Multi-)Factorial models to reduce the number of parameteres :**
  - CAPM with N risky assets : $\Sigma=\beta.\beta' + \Sigma_\varepsilon$ where $\beta= (\beta_1,\ldots, \beta_N)'$ and Σ_ε is diagonal $(\sigma_1^2,\ldots, \sigma_N^2)$
  - Ex : 80 parameters to estimate in the CAC 40 universe

- **Decomposition of risk for a portfolio of risky assets ($\alpha_1,\ldots, \alpha_N$)**
  - $\sigma_P^2 = \beta_P^2 \sigma_F^2$ (systematic or non diversifiable risk) + $(\alpha_1^2 \sigma_1^2 +\ldots+ \alpha_N^2 \sigma_1^2)$ (specific or diversifiable risk)
  - β_P is the relative risk of the portfolio
  - Note : the factor is not replicable by the assets : $\forall$ P : $\sigma_P^2 > \beta_P^2 \sigma_F^2$

- **Problem of factorial models: wrong specification**
  - Ex : Fama and French 3-factor model (market, firm size, style)

- **The problem of estimation remains for any measure of risk**

Limits of volatility
> 3rd limit : non-locally stationary returns

- **Returns may be not stationay even over short periods:**
  - Historical volatility is not an estimator of volatiliy for a (plain vanilla) option
- **Ex-ante volatility rather than historical (ex-post) volatility:**
  - Delta-Linearization of the option : $C_t=\Delta_t.S_t + (1-\Delta_t).B_t$
  - Historical volatility of portfolio $\Delta_k.S_t + (1-\Delta_k).B_t$ to estimate the volatility of the option at time k
  - Correlations can be estimated as well
- **The volatility of the option is not constant over time :**
  - Frequent re-estimation is necessary
- **A very actively-managed fund is similar to a derivative product :**
  - Again, the return of the fund is not stationary
- **Remark : some derivative products cannot be linearized (complex derivative products)**
  - Moreover, their returns are not Gaussian

Limits of volatility
> 4th limit : Non-Gaussian returns

- **For Gaussian returns, expected returns and correlation matrix only parameters of the distribution :**
  - A good estimation of these parameters leads to a good knowledge of risk

- **For stationary and non-Gaussian returns :**
  - Historical volatility is an estimator of volatility…
  - …but volatility is not a good measure of risk (extreme events are underestimated)

- **More complexe parametric laws to fit returns :**
  - Skew-Student distribution to take into account skewness and kurtosis

- **Law for extreme values (Gumbel, Etc.)**
  - Ex : Weekly returns of Euro Stoxx 50 are Gaussian except for years 1987, 1990, 1998, 2000-2002, 2007, 2008-2011.
  - Returns = X (Gaussian) + Y (Gumbel?)

- **VaR as a risk measure :**
  - Threshold value such that the probability of minimum loss over a given time horizon is a given probability
  - For gaussian returns, VaR is proportional to volatitliy :

    VaR 99% 1 day =2.33*standard deviation of daily returns=$2.33*\sigma/(26)^{1/2}$

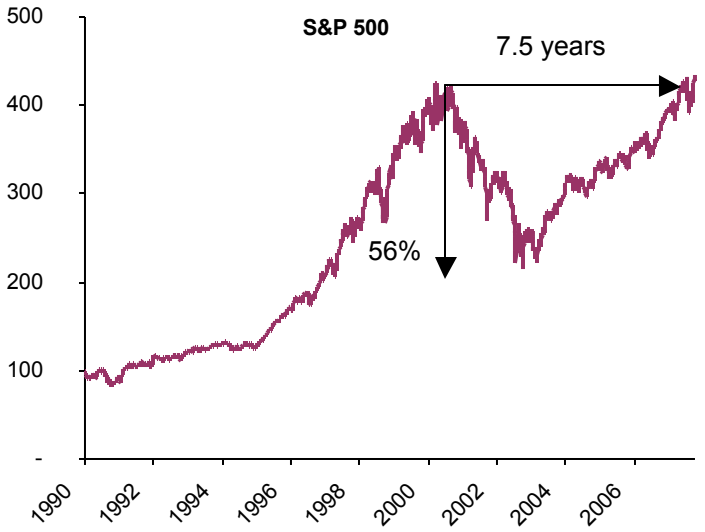
    VaR 99% 1 day =1.45% if volatility of daily (gaussian) returns is 10%

VaR as a risk measure

- **Ex-post (historical) VaR of a portfolio easy to calculate but:**
  - Historical VaR is not an estimator of VaR of a portfolio with derivative products
  - We need to calculate ex-ante VaR
- **Financial regulatory reforms :**
  - Investment Banks (Basel II): VaR 99% 10 days
  - Funds using derivative products (AMF) : VaR 95% 1-week
- **The rule of « maximum loss » applies If the fund manager cannot calculate the VaR of a line :**
  - VaR of portfolio = Maximum loss of the product + remaining VaR
  - It is useless to buy a hedge product in this case
- **Risk department is the only winner of the crisis**

Limits of VaR

- **Problem of estimation :**
  - We need to know the distribution of the returns
  - The more complex the distribution is, the less precise the estimation will be
- **VaR does not measure the extremes of extreme distribution :**
  - Exponential losses
    VaR 99% 1 day = 0.5% but VaR 99,5% 1 day = 50%
  - Introduction to cVaR (conditional) :
    cVaR = expected return of the extreme distribution (losses over VaR)
  - Problem of estimation for cVaR as well
- **Limits of cVaR :**
  - Controlling cVaR does not ensure the safety of principal
  - Expected (excess) returns have to be positive
  - Maximum drawdown and time to recovery as complementary measures

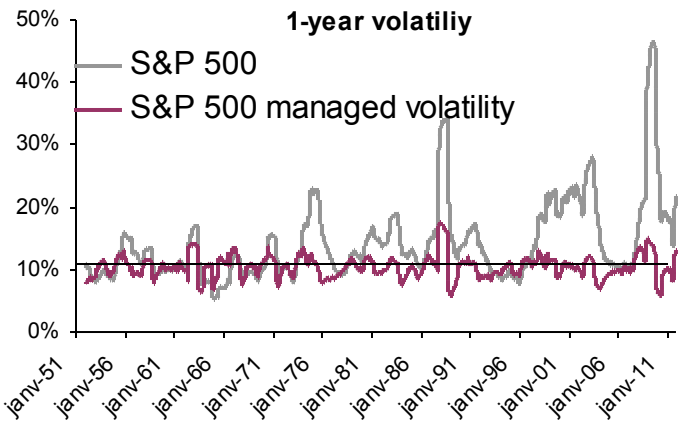


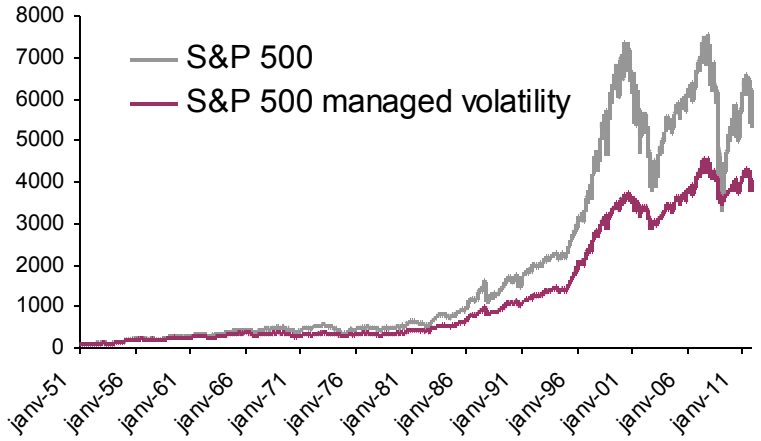
Managing risk
> Managed volatility portfolio

- **Let us suppose the S&P 500 is the market portfolio for US equities market :**
  - An optimal portfolio is made with a portion of S&P 500 and a portion of risk-free asset
  - Target of (ex-ante) volatility set to 10%

- **Ex-post volatility is well controlled :**
  - Mean of 1-year historic (ex-post) volatility = 10.4%
  - 95% of 1-year historic volatility are in the intervall [7.8% ; 13.4%]

- **But ex-post volatility is always higher than ex-ante volatility :**
  - Historical volatility is not an estimator of volatility (the returns are not stationary)
  - We add volatility by managing the exposition to risky assets
  - A solution is to lower the target



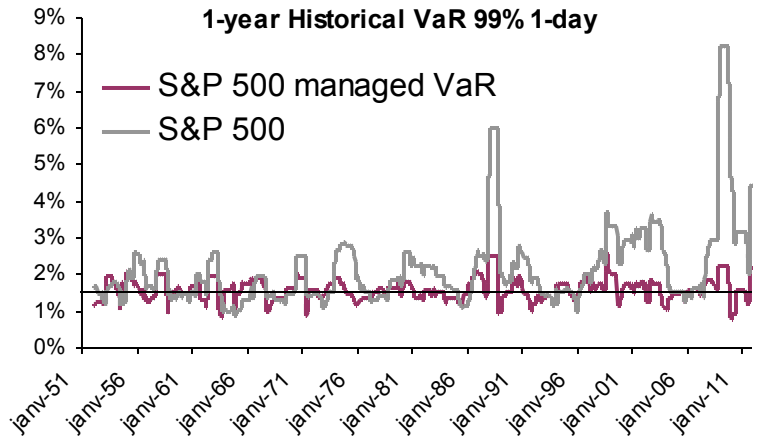


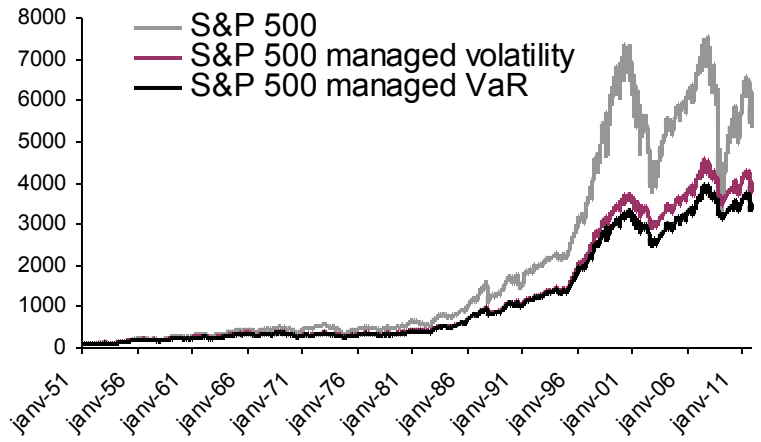
Managing risk
> Managed VaR portfolio

- **Target of VaR 99% 1-day set to 1.45%**

- **Ex-post VaR is well controlled but (again) higher than the ex-ante target :**
  - Mean of 1-year historical VaR 99% 1-day = 1.6%
  - 95% of values in the intervall [1.2% ; 2.1%]
  - Again, historical VaR is not an estimator of VaR

- **Although returns are not Gaussian, the 2 strategies have almost the same characteristics :**
  - Average long-term returns : 6,2% vs. 6%
  - Average 1-year ex-post volatility : 10.4% vs. 10.2%
  - Average 1-year ex-post VaR : 1.59% vs. 1.60%





Conclusion
> Black swan theory (Taleb)

- **All measures need history :**
  - Black swan event : hard-to-predict with major impact
  - Prudential rules are necessary

- **Response to Black Swan events**
  - Diversification : 5/10/40 rule, maximum allowed gross exposure is 200% (UCITS III)
  - Liquidity : daily liquidity, liquid assets, etc.
  - Transparency : no black box

- **Asset Allocation under constraints :**
  - Markowitz under constraints
  - Benchmarked approach (Black-Litterman)
    Pay attention to the efficiency of the benchmark

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